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MA111 - Engineering Mathematics - II
Problem Sheet - 7

First order ODE

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1. Verify (by differentiation and substitution) that $y = ce^{-4x} + 0.35$ is a solution of $y' + 4y = 1.4$. If $y(0) = 2$, find the particular solution.

2. Show (by differentiation and substitution) that $y = cx - c^2$ is a general solution of $(y')^2 - xy' + y = 0$. Show that $y = x^2/4$ is also a solution of this ODE. And this solution cannot be obtained from the general solution. Such a solution is called a **singular solution.**

3. Solve the following IVP.
$$xy' = y + 4x^5 \cos^2\left(\frac{y}{x}\right), \; y(2) = 0. \quad (\text{set } \frac{y}{x} = u).$$
Answer: $y = x \arctan(x^4 - 16)$.

4. Find a general solution of
$$y' = (4x + y + 1)^2, y(0) = 2. \text{ (set } v = 4x + y + 1)$$
Answer: $4x + y + 1 = 2\tan(2x + c)$.

5. Solve
$$(-y/x^2 + 2\cos 2x)\, dx + (1/x - 2\sin 2y)\, dy = 0.$$
Answer: $y/x + \sin 2x + \cos 2y = c$.

6. Solve
$$(e^{(x+y)} - y)\, dx + (xe^{x+y} + 1)\, dy = 0.$$
Answer: $xe^y + ye^{-x} = c$.

7. Under what conditions for the constants a, b, k, l is
$$(ax + by)dx + (kx + ly)dy = 0$$
exact? Solve the exact ODE.
Answer: $b = k,\; ax^2 + 2kxy + ly^2 = c$

8. Find the general solution of following IVP's. If an initial condition is given, find also the corresponding particular solution.

   (a) $x^2y' + 3xy = 1/x,\; y(1) = -1$.
   Answer: General solution: $y = x^{-3}(x + c)$
   Particular solution: $y = x^{-2} - 2x^{-3}$

(b) $y' + 4x^2y = (4x^2 - x)e^{-x^2/2}$
Answer: $y = ce^{-4x^3/3} + e^{-x^2/2}$

(c) $y' + x^2y = (e^{-x^3}\sinh x)/(3y^2)$
Answer: $y = \left\{e^{-x^3}(\cosh x + c)\right\}^{1/3}$

(d) $y' \sin 2y + x\cos 2y = 2x$ (Reduce to a linear equation by the substitution $z = \cos 2y$, or use separation of variables)
Answer: $y = \frac{1}{2}(\arccos(2 + ce^{x^2})$

9. Show that the sum of a solution of the non-homogeneous linear ODEs ((a) $y' + p(x)y = r(x)$) and a solution of the homogeneous linear ODEs ((b) $y' + p(x)y = 0$) is a solution of (a).

10. Apply Picard's method to calculate y_1, y_2, y_3, y_4 for the IVPs given below (i.e., perform 4 iterations).

(a) $y' = 2x - y, y(0) = 1$.
(b) $y' = y^2, y(0) = 1$.

11. Does the following IVP have a unique solution on some interval with center 0?

$$y' = x^2 + y^2,\ y(0) = 0 \text{ on } \{(x,y) : |x| \leq 1; |y| \leq 1\}?$$

12. Let $f(x,y) = y^{\frac{2}{3}}$ defined on $R : |x| \leq 2; |y| \leq 2$. Does f satisfy Lipschitz condition in the variable y on R?

13. Can two solution curves of the same ODE have a common point in a rectangle in which the assumptions of Theorem 2 of Section 1.7 of your course textbook are satisfied? Explain.
Answer: No.

14. When does the IVP $xy' = y + 3x^2,\ y(x_0) = y_0$ have no solution, precisely one solution, and more than one solution?

Answer: No solution if $x_0 = 0,\ y_0 \neq 0$.
One solution if $x_0 \neq 0$.
Infinitely many solutions if $x_0 = 0,\ y_0 = 0$.

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